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Validity of a modified Born approximation for a pulsed plane wave in acoustic scattering problems

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Abstract
The validity domain of a modified Born approximation (MBA) has been examined for the scattering of a pulsed plane wave (PPW). This new approximation has been compared with exact results and also with the conventional Born approximation (BA). Comparisons have been made for the scattering by a homogeneous sphere. Error charts have been presented for various scatterer sizes and acoustic properties for forward as well as back scattering. Pulse width has also been varied. Our study shows that the modified Born approximation is generally preferable to the conventional Born approximation in the forward direction. In the backward direction both approximations have almost similar kind of validity domain. These observations are important in view of the fact that the Born approximation has been widely used in acoustic scattering problems.

1. Introduction

Ultrasound scattering has found important applications in many disciplines of science, engineering and medicine. The applications vary from flaw detection and quality control in industrial processes to underwater scattering in oceans to a variety of medical uses. These applications require use of either single scattering or multiple scattering theories for the analysis of measurements. In either case one requires knowledge of the underlying single scattering phenomenon. Unfortunately, it is not always possible to obtain exact single scattering solutions and hence it is customary to employ approximation methods. One such approximation that has been employed extensively is the Born approximation (Morse and Ingard 1968). This approximation is good for those scatterers whose sizes are small compared to the wavelength of the incident wave and whose density and compressibility differ only slightly from those of the surrounding medium. The approximation leads to formulae that are easy to compute and which yield reasonably accurate results over the entire angular domain.

In a recent publication (Sharma and Saha 2004), we proposed the use of a modified BA and examined its validity for the problem of scattering of a plane acoustic wave. This new approximation is an analogue of an approximation previously used successfully in the context of light scattering by small particles (Shimizu 1983, Sharma and Somerford 1988). Our studies in Sharma and Saha (2004) clearly demonstrated that MBA for plane wave acoustic scattering results in greater accuracy and considerable enlargement of the validity domain as compared to that of the conventional BA for forward scattering.

One of the limitations of our earlier study was that it was mainly concerned with the scattering of single frequency plane waves. However, one may be interested in the scattering of short duration pulses for its wide range of applications. The plane wave study, therefore, needs to be extended to the scattering of a pulse. In this paper we address ourselves to this question. Thus, we examine the validity of MBA vis-a-vis validity domain of BA for the case of a pulsed plane wave. As the accuracy of BA or MBA for a plane wave is frequency dependent, the validity of these approximations for a pulse may depend on the frequency content of the pulse in general. The purpose here is to estimate the errors involved in BA and MBA for typical pulse and scatterer sizes encountered in practical applications where BA is generally employed.

The second limitation of our earlier study was that it concentrated primarily on examining the validity of these approximations for forward scattering. Present study is more general as both forward as well as back scatterings have been examined. The study of validity of approximations for back scattering is important since most biomedical applications employ back scattering because of its suitability for *in vivo* studies (Lzzi and Coleman 1983, Insana *et al* 1990, Shung and Thieme 1993). Nevertheless, non backward scattering finds biomedical applications in studies such as in ultrasonic diffraction tomography (Miyashita and Honda 2000) and red blood corpuscles (RBC) studies (Ishimaru 2002).

The organization of the paper is as follows. In section 2, we give relevant scattering formulae for exact and for approximate methods for the scattering of a plane wave by a sphere. In section 3, we obtain corresponding quantities for a pulsed plane wave. Section 4 is devoted to numerical comparison. Theoretically BA and MBA have the same validity domain. Hence to delineate their validity domains one has to take resort to numerical comparisons. The model of scattering by a homogeneous sphere is employed for this purpose because exact solutions can be easily computed for this shape. Conclusions from this study are presented in section 5.

2. Scattering of a plane wave by a homogeneous sphere

The exact scattering amplitude or the angle distribution factor is given by (Morse and Ingard 1968)

$$\Phi_{\mathrm{ex}}(k, \theta) = \frac{\mathrm{i}}{k} \sum_m (2m+1) b_m P_m(\cos\theta), \tag{1}$$

where

$$b_m = \frac{j'_m(x) j_m(y) - \alpha j_m(x) j'_m(y)}{h'_m(x) j_m(y) - \alpha h_m(x) j'_m(y)}, \tag{2}$$

with $x = ka$, $y = nx = nka$, $n = k_e/k = c/c_e$, $\alpha = n\rho/\rho_e$ and j_m and h_m are the spherical Bessel and Hankel functions of order m and the prime denotes differentiation with respect to the argument. Here a is the size of the scatterer, k and c are the wave number and velocity of the sound wave in the homogeneous loss-less medium, k_e and c_e are the corresponding quantities of the scattering region. The density and compressibility are designated by ρ and κ respectively of the surrounding medium whereas ρ_e and κ_e are same quantities within the object. The scattering angle θ is the angle between the direction of observer $\mathbf{k_s}$ and the direction of incident wave $\mathbf{k}$. The subscript ex corresponds to the exact solution. For single frequency plane wave scattering differential scattering cross section ($|\Phi(k,\theta)|^2$), which is the square modulus of scattering amplitude, is always a quantity of interest. The scattering amplitude $\Phi(k,\theta)$ is related to the scattered pressure field $p_{\mathrm{ex}}^{(s)}(\mathbf{r},t)$ in the asymptotic region $r\to\infty$ via the relation,

$$p_{\mathrm{ex}}^{(s)}(\mathbf{r},t)=\Phi_{\mathrm{ex}}(k,\theta)\frac{\mathrm{e}^{\mathrm{i}k(r-ct)}}{r}. \tag{3}$$

The superscript s indicates the scattered wave. Analogous expressions are valid for approximate pressure fields.

In the Born approximation the scattering amplitude may be expressed as (Morse and Ingard 1968)

$$\Phi_b(k,\theta)=\frac{x^2}{q}[\gamma_\kappa+\gamma_\rho\cos\theta]j_1(qa), \tag{4}$$

where, $\gamma_\kappa=\frac{\kappa_e-\kappa}{\kappa}$ and $\gamma_\rho=\frac{\rho_e-\rho}{\rho_e}$ are the normalized compressibility and density mismatch parameters respectively and a is the radius of the particle. In equation (4) $\mathbf{q}=\mathbf{k_s}-\mathbf{k}$ is the wave-vector transfer with magnitude $2k\sin(\theta/2)$ and j_1 is the spherical Bessel function of order unity. The suffix b stands for the Born approximation. Theoretically, the conditions for the validity of BA are,

$$|\gamma_\kappa|\ll 1 \qquad |\gamma_\rho|\ll 1, \tag{5a}$$

$$x|\gamma_\kappa|\ll 1 \qquad x|\gamma_\rho|\ll 1. \tag{5b}$$

In this approximate method it is assumed that the pressure field inside the scatterer is nothing but the incident field ($\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$) and hence the wave number within the scattering region remains same as that of the incident wave propagating through the ambient medium. This assumption is valid only when the conditions (5a) and (5b) are maintained for the scatterer. The scatterer is then said to be a weak scatterer.

Many workers (Shimizu 1983, Sharma and Somerford 1988) successfully employed a modified version of BA to explain the scattering of light by small particles. In this modification they took the field within the scatterer as $\mathrm{e}^{\mathrm{i}n\mathbf{k}\cdot\mathbf{r}}$ instead of $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$ where $nk=k_e$. This makes the fields inside the scatterer dependent on the properties of the scatterer. This modification enlarges the validity domain of BA in acoustical scattering too as shown in detail in Sharma and Saha (2004). The scattering amplitude in the modified Born approximation is formally similar to equation (4) and can be written as

$$\Phi_{\mathrm{mb}}(k,\theta)=\frac{x^2}{R}[\gamma_\kappa+n\gamma_\rho\cos\theta]j_1(Ra), \tag{6}$$

where $\mathbf{R}=n\mathbf{k_s}-\mathbf{k}$ and its magnitude is $R=k\sqrt{1+n^2-2n\cos\theta}$. The subscript mb corresponds to the modified Born approximation. Note that MBA does not introduce any extra complications in the formula for the angular scattering function. Theoretically, the conditions for validity of MBA are also given by equation (5).

3. Formulae for scattering of a pulsed plane wave

Since a pulse can be represented as a linear superposition of plane waves, the asymptotic scattered pressure field can be expressed as

$$p^{(s)}(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(k)\Phi(k, \theta) \frac{\mathrm{e}^{\mathrm{i}k(r-ct)}}{r} \,\mathrm{d}k, \tag{7}$$

where $g(k)$ is the weight factor for spatial frequency k and the form of $g(k)$ depends upon the shape of the incident pulse. A real pulse emitted by a medical ultrasound transducer can be approximated as a Gaussian wave packet (Szabo 2004, Tobocman *et al* 2002, Kharin *et al* 2003) though in time domain the tailing edge falls slowly comparative to the leading edge. Thus, for our purpose in this paper we assume that the incident pulse is a normalized Gaussian wave packet. At time $t = 0$, the packet is given by

$$p^{(\mathrm{in})}(z) = \left(\frac{1}{\pi\sigma^2}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{(z+\mu)^2}{2\sigma^2}} \,\mathrm{e}^{\mathrm{i}k_0(z+\mu)}. \tag{8}$$

This is a Gaussian modulated plane wave of wave number k_0 and propagating in the positive z direction. The spread of this wave packet in space is σ. At time $t = 0$ the centre of the wave packet is located at $z = -\mu$ or in other words the pulse is launched from $z = -\mu$. The superscript (in) refers to the incident wave. The Fourier transform of incident pressure field gives the frequency distribution of the pulse as

$$g(k) = \tilde{p}^{(\mathrm{in})}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} p^{(\mathrm{in})}(z)\,\mathrm{e}^{-\mathrm{i}kz} \,\mathrm{d}z. \tag{9}$$

Substitution of equation (8) into equation (9) yields

$$\tilde{p}^{(\mathrm{in})}(k) = \left(\frac{1}{\pi\sigma^2}\right)^{\frac{1}{4}} \sigma\,\mathrm{e}^{-\frac{(k-k_0)^2\sigma^2}{2}} \,\mathrm{e}^{\mathrm{i}k\mu}. \tag{10}$$

The time evolution of the incident pressure wave packet may be expressed as

$$p^{(\mathrm{in})}(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{p}^{(\mathrm{in})}(k)\,\mathrm{e}^{\mathrm{i}k(z-ct)} \,\mathrm{d}k, \tag{11}$$

and the associated particle velocity is given by

$$u^{(\mathrm{in})}(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{p}^{(\mathrm{in})}(k)}{\rho c} \,\mathrm{e}^{\mathrm{i}k(z-ct)} \,\mathrm{d}k, \tag{12}$$

where ρ is the density of the medium.

The pulse intensity integral (PII) for the incident wave packet at a particular point z is defined (American Institute of Ultrasound in Medicine 1992) as the time integral of the instantaneous intensity i.e.,

$$\mathrm{PII}^{(\mathrm{in})}(z) = \frac{1}{2} \int_{-\infty}^{\infty} p^{(\mathrm{in})}(z, t) u^{(\mathrm{in})*}(z, t) \,\mathrm{d}t, \tag{13}$$

where $*$ denotes the complex conjugate of the function. Though the limits of integration in equation (13) are from $-\infty$ to ∞, actually most energy is contained within a time interval which is known as pulse duration. In this time interval, the strength of the signal is sufficient to excite the receiver. Beyond this the signal is less than the lower cut-off of the detector and indistinguishable from the background noise. Pulse duration is defined as the period between times when the pulse amplitudes become -20 dB with respect to the maximum amplitude

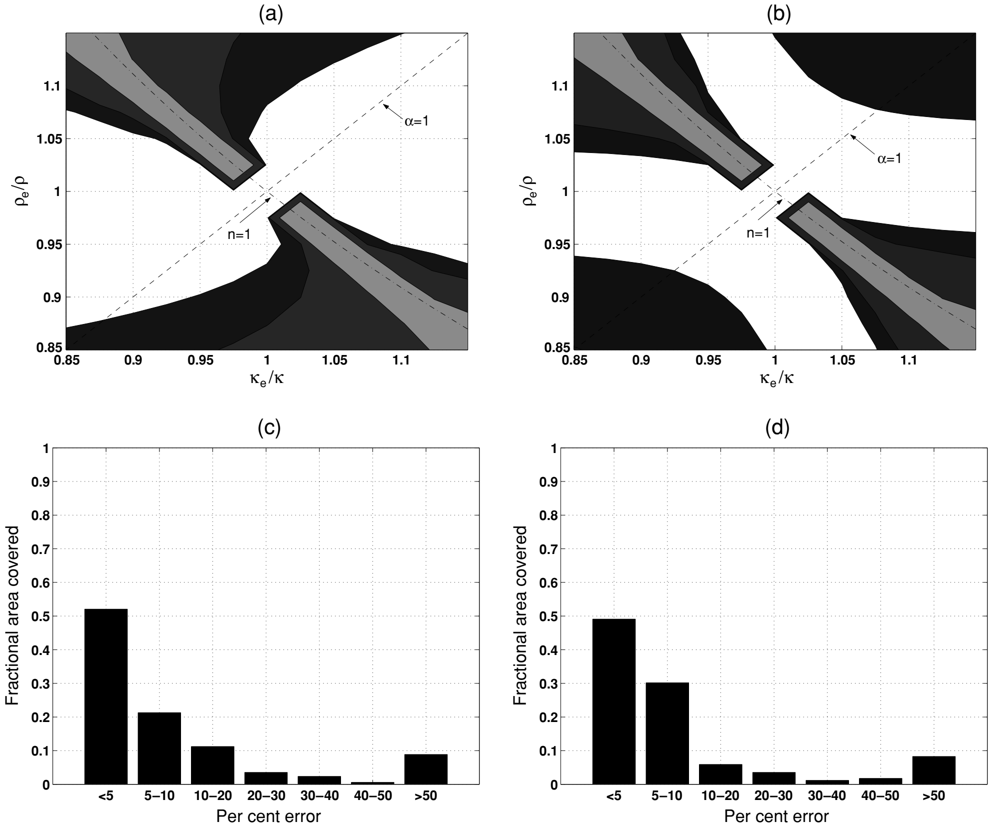

Figure 1. (a) PII error contour charts for BA in the forward direction for $x = 0.1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) Same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

(Raum and O'Brien 1997). The explicit form of $\mathrm{PII}^{(\mathrm{in})}$ for the incident wave packet can be obtained by inserting equations (11) and (12) into equation (13). The $\mathrm{PII}^{(\mathrm{in})}$ becomes

$$\mathrm{PII}^{(\mathrm{in})}(z) = \frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}t\int_{-\infty}^{\infty}\tilde{p}^{(\mathrm{in})}(k)\,\mathrm{e}^{\mathrm{i}k(z-ct)}\,\mathrm{d}k\int_{-\infty}^{\infty}\frac{\tilde{p}^{(\mathrm{in})*}(k')}{\rho c}\,\mathrm{e}^{-\mathrm{i}k'(z-ct)}\,\mathrm{d}k'. \tag{14}$$

Integrations in the above equation can be carried out to yield

$$\mathrm{PII}^{(\mathrm{in})}(z) = \frac{1}{2\rho c}\int_{-\infty}^{\infty}|\tilde{p}^{(\mathrm{in})}(k)|^2\,\mathrm{d}k = \frac{1}{2\rho c}. \tag{15}$$

Integrand in equation (15) is the intensity for each plane wave. The scattered pressure field in the asymptotic region and the corresponding particle velocity in the radial direction are given by the following relations,

$$p_{\mathrm{ex}}^{(s)}(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{p}^{(\mathrm{in})}(k)\Phi_{\mathrm{ex}}(k,\theta)\frac{\mathrm{e}^{\mathrm{i}k(r-ct)}}{r}\,\mathrm{d}k, \tag{16}$$

and

$$u_{\mathrm{ex}}^{(s)}(\mathbf{r},t) = \frac{1}{\sqrt{2\pi}}\frac{1}{\rho c}\int_{-\infty}^{\infty}\tilde{p}^{(\mathrm{in})}(k)\Phi_{\mathrm{ex}}(k,\theta)\frac{\mathrm{e}^{\mathrm{i}k(r-ct)}}{r}\,\mathrm{d}k. \tag{17}$$

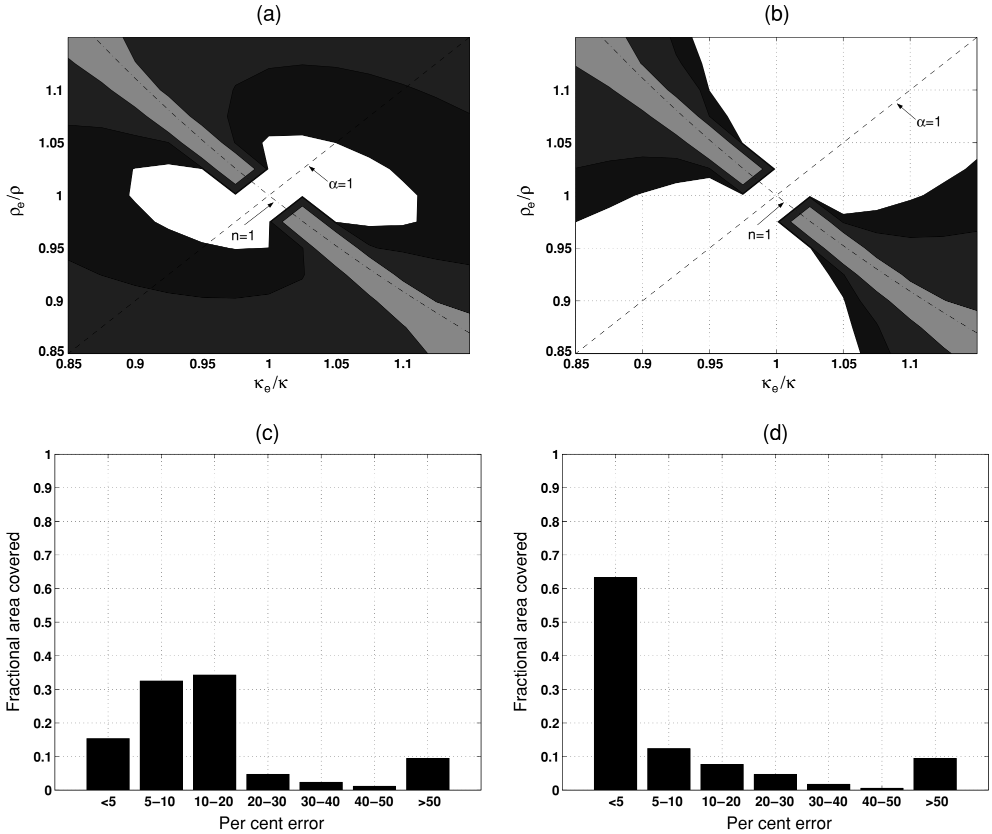

Figure 2. PII error contour charts for BA in the forward direction for $x = 1$ and the narrow bandwidth pulse (a) in the BA, (b) in the MBA. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error.

We can also derive analogous expressions for the approximations. In general $\Phi(k, \theta)$ is a complex quantity and hence $p_{\text{ex}}^{(s)}(\mathbf{r}, t)$ retains the information of phase change that has been taken place during the interaction of the plane waves with the scatterer. In the asymptotic region, the phase difference between the pressure and the particle velocity can be neglected and the radial component of the pulse intensity integral at a particular radial point $(\mathbf{r})$ for the scattered pressure is given by

$$\text{PII}_{\text{ex}}^{(s)}(\mathbf{r}) = \frac{\text{PII}^{(\text{in})}}{r^2} \int_{-\infty}^{\infty} |\tilde{p}^{(\text{in})}(k)\Phi_{\text{ex}}(k, \theta)|^2 \, \mathrm{d}k. \tag{18}$$

The integrand in equation (18) gives the differential scattering cross section for a plane whose wave number is k. Similar relations hold for approximations too. Note that $\text{PII}^{(s)}(\mathbf{r})$ does not contain any phase information.

For scanned modes (A mode, B mode) pressure field distribution or more precisely maximum positive peak pressure (PP_{max}) is a quantity of interest whereas for unscanned modes (pulsed Doppler mode, M mode) PII is one of the quantities of interest. To compare approximation methods we follow both the methodologies (PP_{max} and PII).

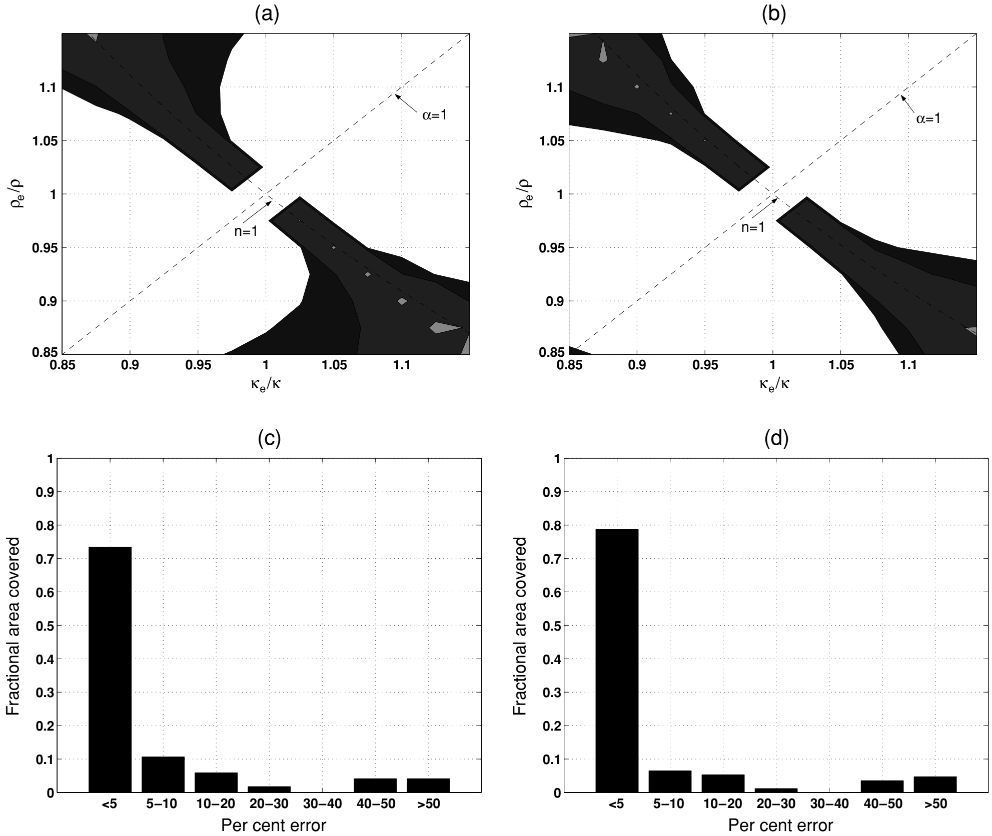

Figure 3. (a) PP_{max} error contour charts for BA in the forward direction for $x = 0.1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) Same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

4. Numerical comparison

This section gives results of a quantitative study of validity of BA and MBA. The ambient medium is chosen as a biomedical tissue-like medium. The acoustic parameters, like density and compressibility of the scattering region, are varied within $\pm 15\%$ with respect to those parameters of the surrounding homogeneous medium. Hence the scatterer can be treated as a weak scatterer and the approximation methods (BA and MBA) may be expected to work faithfully for the description of single particle scattering phenomenon. This variation corresponds to typical variations in medical problems. For example γ_ρ and γ_κ for a red blood cell in plasma are 0.0695 and 0.166 respectively (Ishimaru 2002). For a plane wave of frequency 5 MHz, the wavelength is 308 μm for a typical tissue-like medium with velocity of 1540 m s^{-1}. If the equivalent sphere size of the RBC is taken to be 5.5 μm, the x value can be calculated to be $x = 0.056$. For larger scatterers such as muscle tissues, cells can be as large as 40 μm giving an $x = 0.41$. The values for x can be quite large for scatterers such as tumours (Hinders *et al* 1992). Thus, we have chosen to examine the validity of approximations for scatterers in the size range $x = 0.01$ to $x = 5.0$ with respect to the central frequency of the pulse. Two different incident pulses have been taken for numerical calculations. Bandwidth

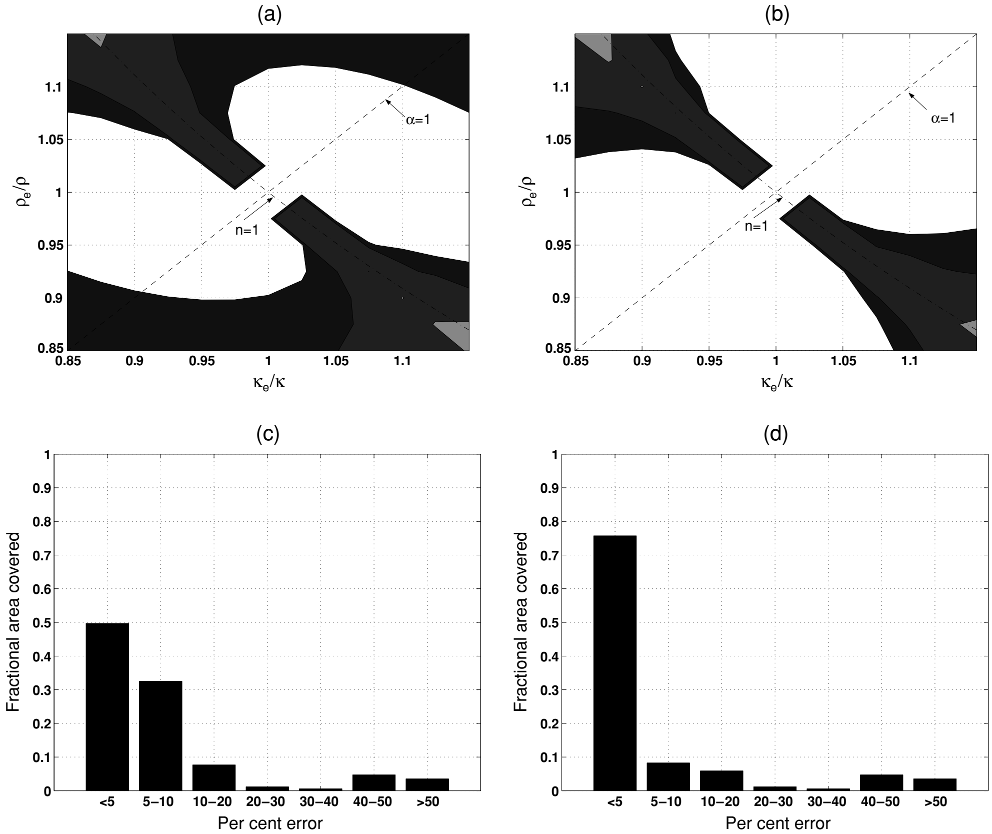

Figure 4. PP_{max} error contour charts for BA in the forward direction for $x = 1$ and the narrow bandwidth pulse (a) in the BA, (b) in the MBA. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error.

(−6 dB) of the first pulse is 1.48 MHz and its duration is ≈547 ns. The bandwidth (BW) of the second pulse is 7.91 MHz with ≈69 ns pulse duration. While the bandwidth of the first pulse is close to a typical pulse launched by the clinical transducers, the second pulse has been employed to study the bandwidth dependence of the approximations. 5 MHz is the centre frequency for both of them. This typical value of centre frequency is taken on the basis that most of the diagnostic ultrasound instruments operate within the frequency range 2–10 MHz.

To compare the errors in BA and MBA we define per cent errors in PII as,

$$\text{PII per cent error} = \frac{\left|\text{PII}_{\text{ex}}^{(s)} - \text{PII}_{\text{approx}}^{(s)}\right|}{\text{PII}_{\text{ex}}^{(s)}} \times 100, \tag{19}$$

where $\text{PII}_{\text{ex}}^{(s)}$ and $\text{PII}_{\text{approx}}^{(s)}$ are the pulse intensity integrals of the scattered pulse in the exact method and in the approximate method respectively. Similarly we define PP_{max} per cent errors for an approximation method as

$$\text{PP}_{\text{max}} \text{ per cent error} = \frac{\left|\max\left[p_{\text{ex}}^{(s)}\right] - \max\left[p_{\text{approx}}^{(s)}\right]\right|}{\max\left[p_{\text{ex}}^{(s)}\right]} \times 100, \tag{20}$$

where $\max\left[p_{\text{ex}}^{(s)}\right]$ is the maximum value of positive peak pressure of scattered pressure pulse.

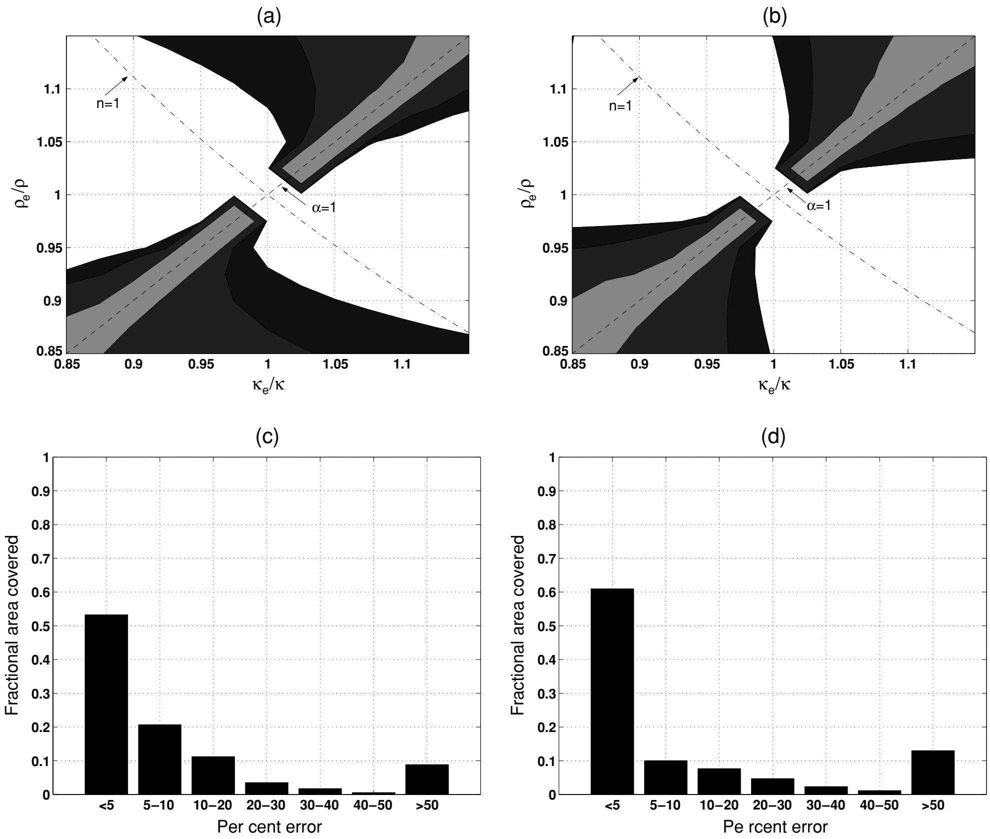

Figure 5. (a) PII error contour charts for BA in the backward direction for $x = 0.1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) Same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

Table 1. PII error.

		Fractional area covered by the error							
		Error <5%				Error <10%			
		Narrow BW		Wide BW		Narrow BW		Wide BW	
Direction	Size	BA	MBA	BA	MBA	BA	MBA	BA	MBA
Forward	0.1	0.52	0.49	0.52	0.49	0.73	0.79	0.72	0.79
	1.0	0.15	0.63	0.19	0.50	0.48	0.75	0.42	0.82
	5.0	0.15	0.31	0.19	0.27	0.30	0.57	0.37	0.52
Backward	0.1	0.53	0.60	0.53	0.61	0.74	0.71	0.74	0.71
	1.0	0.43	0.49	0.56	0.43	0.74	0.62	0.82	0.60

Figures 1 and 2 show typical comparison of errors in PII for BA and MBA for $x = 0.1$ and for $x = 1.0$ respectively for forward scattering for the narrow bandwidth pulse. The numerical values of areas covered by the regions with errors less than 5% and 10% are shown in table 1 for $x = 0.1$, 1.0 and 5.0. It can be seen from table 1 that while BA and MBA have

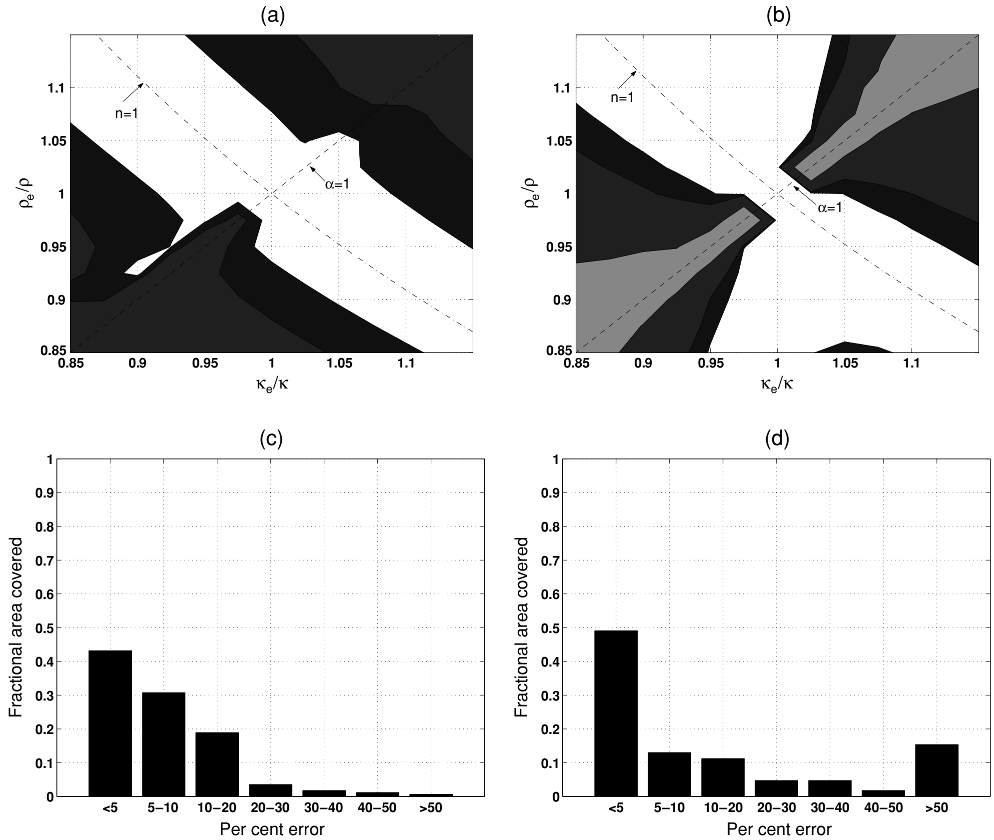

Figure 6. (a) PII error contour charts for BA in the backward direction for $x = 1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

Table 2. PP_{max} error.

		Fractional area covered by the error							
		Error <5%				Error <10%			
		Narrow BW		Wide BW		Narrow BW		Wide BW	
Direction	Size	BA	MBA	BA	MBA	BA	MBA	BA	MBA
Forward	0.1	0.73	0.78	0.73	0.78	0.84	0.85	0.84	0.84
	1.0	0.49	0.75	0.47	0.88	0.82	0.84	0.91	0.97
	5.0	0.30	0.58	0.36	0.53	0.66	0.96	0.65	0.92
Backward	0.1	0.74	0.71	0.74	0.71	0.85	0.77	0.84	0.77
	1.0	0.74	0.61	0.77	0.66	0.92	0.73	0.90	0.77

similar validity domain for small size scatterers, MBA distinctly has a larger validity domain in comparison to BA for large scatters. One can observe the same trend for a wide band pulse too. Results for typical comparison of errors in PP_{max} for BA and MBA for $x = 0.1$ and

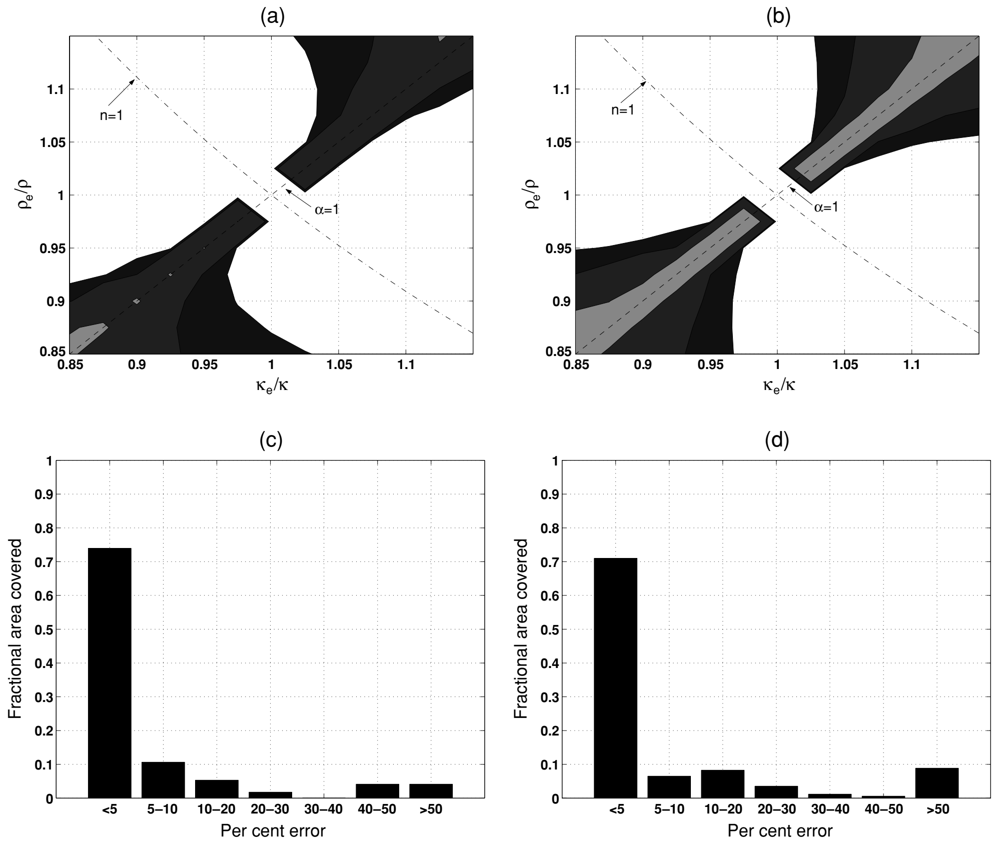

Figure 7. (a) PP_{max} error contour charts for BA in the backward direction for $x = 0.1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) Same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

for $x = 1.0$ respectively for forward scattering for the narrow bandwidth pulse are shown in figures 3 and 4. The numerical values of regions with errors less than 5% and 10% are shown in table 2 for $x = 0.1$, 1.0 and 5.0. Results are similar to those obtained for PII. The same holds for wide band pulse too. Clearly, MBA is almost always preferable to BA for forward scattering.

Figures 5 and 6 show per cent errors in BA and MBA for $x = 0.1$ and $x = 1.0$ respectively for PII for the back scattering of narrow bandwidth pulse. The numerical values of areas for regions with errors less than 5% and 10% are shown in table 1 for $x = 0.1$ and 1.0. Corresponding values for a wide band pulse are also shown in the same table. It is interesting to note that although, BA generally appears preferable to MBA, there are regions where the use of MBA is clearly advantageous. For example, a glance at table 1 shows that for $x = 0.1$ (narrow as well as wide band case for error <5%) MBA has a wider domain of validity in comparison to BA. Since there are no extra complications in the use of MBA, the use of MBA is preferable for such particles even for back scattering analysis. Clearly, the accuracy of BA and MBA should be compared for the region of interest before either approximation is used

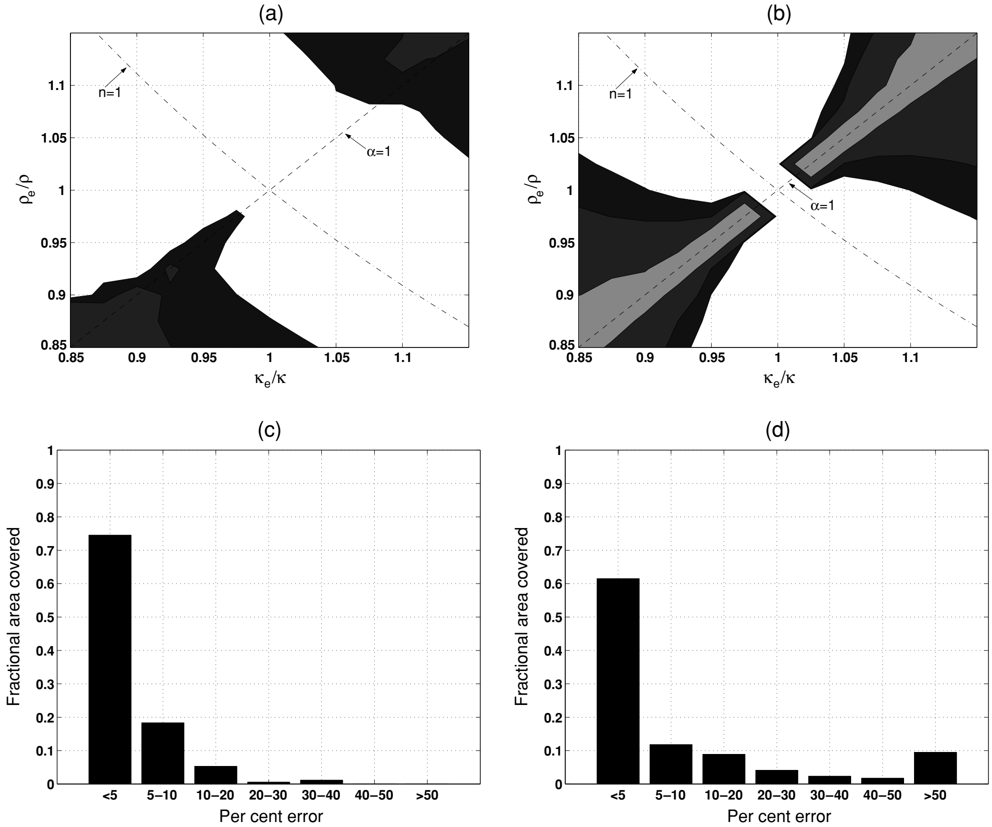

Figure 8. (a) PP_{max} error contour charts for BA in the backward direction for $x = 1$ and the narrow bandwidth pulse. White area: <5 per cent error, black area: <10 per cent error, less black area: <50 per cent error and least black area: >50 per cent error. (b) same as (a) but for MBA. (c) Bar diagram for (a). (d) Bar diagram for (b).

for the back scattering. In contrast for PP_{max} however, BA appears to be always preferable for back scattering. This can be seen from figures 7 and 8 and table 2.

A look at tables 1 and 2 also reveals that the change of bandwidth does not result in significant changes in the validity domains of the approximations. Less than 15% change in the area with respect to the total area has been observed here. Another interesting feature of the approximations that can be noted from figures 1–8 is that while the approximations are good for forward scattering in the limit $\alpha \to 1$, for back scattering the approximations should be looked upon as $n \to 1$ approximation. That is for forward scattering the approximations are good when the relative acoustic impedance goes to 1. On the other hand, the approximations are good for back scattering if the relative velocity between the scatterer and the surrounding medium approaches to 1.

5. Conclusion

In this paper we have drawn attention of workers in the field of ultrasound to an approximation that takes into account a simple modification in the Born approximation. We have evaluated the validity of BA as well as MBA for the scattering of pulsed plane wave by a homogeneous sphere. It has been shown that MBA can be extremely useful for predicting the ultrasound scattering by weak scatterers.

The approximations have been examined for the domain of x values ranging from 0.01 to 5.0. The mismatch of the density and compressibility between the scatterer and the surrounding medium has been taken to be up to 15%. In this regime n and α values also vary within 15%. These mismatches are typical for problems encountered in medical systems. For example, (i) acoustic properties of tissues in brain, kidney, liver and various types of muscle differ by less than 15% from those of water (Christensen 1988), (ii) Hinders *et al* (1992) have taken 10% variation in density and wave speed with respect to surrounding tissue for calculations of scattering by a spherical tumour. Representative results of our study have been presented as error contour charts and error bar charts for forward as well as back scattering. Corresponding numerical values of errors are displayed in the tables. The quantities for which the errors are compared are the pulse intensity integral and the maximum positive peak of the scattered pressure pulse. Typical narrow band and a wide band pulses have been considered.

Following conclusions may be drawn from this study of the validity of BA and MBA for scattering of a pulse by a weak scatterer. (i) For forward scattering, MBA is always preferable in comparison to BA (for PII as well as PP_{max}). (ii) For back scattering, BA is always preferable in comparison to MBA for PP_{max} only. For PII, although BA seems to be generally preferable, there are regions where MBA clearly has an advantage. For example, for small particles, there are regions, MBA distinctly has a wider domain of applicability in comparison to BA even for back scattering. Thus when back scattering is used in conjunction with PII, it would be desirable to compare the two approximations in the region of interest and employ one which is more accurate for those set of parameters. (iii) Change of bandwidth does not result in significant change in the validity domains of the approximations. (iv) For forward scattering both approximations should be looked upon as $\alpha \rightarrow 1$ approximation. On the other hand, for back scattering the approximations should be thought as $n \rightarrow 1$ approximation.

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