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Improved Born approximations for acoustic wave back scattering in tissue medium

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The density and compressibility fluctuations in a tissue medium act as scattering centres for an incoming ultrasound wave and accordingly the scattered wave carries a significant amount of information about the tissue medium. Thus, in principle, it is possible to characterize the tissue medium by analyzing the scattered component of the wave-in particular the backscattered component. In the method developed so far to achieve this end, the Born approximation is almost exclusively employed to describe the acoustic scattering process. In this paper, we propose two improved forms of the Born approximation which may be used in place of the conventional Born approximation. The validity domain of these approximations has been studied in case of exactly soluble models. The potential of this approach for solving tissue scattering problems is being examined.

INTRODUCTION

The interaction between a biomedical tissue and ultrasound sound involves complicated process and requires sophisticated models for its description. In addition to reflection and refraction at the tissue boundaries, the scattering by tissue inhomogeneities also play an important role. Conventional ultrasound imaging systems produce a gray scale image by converting the intensity of the echo signal into gray value[1]. This echo signal also contains the scattered signals from the inhomogeneities. For imaging purpose, this part of the signal is generally regarded as unwanted noise. However, the analysis of the scattered signal has been shown to be very useful for the purpose of tissue characterization. By analysing the back scattered signal, many workers estimated tissue characteristics such as the size of the scatterers, their number density and scattering strength[2].

In these models the tissue is treated as an inhomogeneous medium consisting of scattering centres of sizes small in comparison to the wavelength of the incident ultrasound wave. The scatterers are assumed to be randomly distributed. The scattering from a single scatterer in these models is assumed to be described in the framework of the Born approximation.

Recently, Tobocman *et al* have proposed a one dimensional scattering model for characterization of layered tissue by profiling its acoustic impedance. The layered tissue model has been applied to biomedical tissues such as the colon of pig. This approach is also based on the Born approximation. Clearly the accuracy of the approach is dependent on the accuracy of the Born approximation.

It is surprising that although the Born approximation is employed so widely in tissue scattering problems, it appears that no attempt has been made to study the accuracy of the Born approximation quantitatively for parameters of scatterers pertinent to biomedical tissues. Nor has there been any attempt to obtain improved approximations. Our aim in this paper is two fold, (i) We examine the validity of the Born approximation in three dimensional as well as one dimensional back scattering and, (ii) We design new approximation methods and compare their accuracy with the Born approximation. For these purposes, we consider (i) the exactly soluble case of scattering of ultrasound by a homogeneous sphere in three dimensions and (ii) the one dimensional exactly soluble model for the layered tissue.

The organization of this paper is as follows. In section 2, we consider three dimensional back scattering from a solid sphere and give relevant exact and approximate formulae. Section 3 is devoted to exact and approximate formulae for scattering by an one dimensional homogeneous layer. In section 4, we present numerical comparisons of approximated formulae with exact results for both three dimensional as well as one dimensional models. Finally, we conclude by summarizing results in section 5.

SCATTERING FORMULAE IN THREE DIMENSIONS

In a homogeneous loss-less medium, waves can propagate indefinitely. However, due to the fluctuations of density and compressibility, the real tissue medium is inhomogeneous. The exact scattering solution of scattering amplitude for a single scatterer can be found by solving the wave equation both inside and outside of the inhomogeneity and then matching them at the boundary. The exact solution exists for very few simple shapes of scatterers. The scattering amplitude or the angle (θ) distribution factor can be exactly calculated[4] for the case of a homogeneous spherical scatterer partial wave analysis in terms of Legendre polynomials $P_m(\cos\theta)$ and is given by,

$$\Phi_{ex}(\theta)=\frac{i}{k}\sum_{m}(2m+1)b_m P_m(\cos\theta) \tag{1}$$

where

$$b_m=\frac{j'_m(x)j_m(y)-\alpha j_m(x)j'_m(y)}{h'_m(x)j_m(y)-\alpha h_m(x)j'_m(y)} \tag{2}$$

with x = ka, y = nx, $n = k_e/k = c/c_e$, α = nρ/ρe and j_m and h_m are the spherical Bessel and Hankel functions of order m and the prime denotes differentiation with respect to the argument. Here α is the size of the scatterer, k and c are the wave number and velocity of the sound wave in the homogeneous loss-less medium, the k_e and c_e are the corresponding quantities of the medium inside the scatterer. The density and compressibility are designated by ρ and k respectively of the surrounding medium where as ρ_e and k_e are same quantities within the object. Here the scattering angle i.e. the angle between the direction of observer k_s and the direction of incident wave k is θ. The subscript ex corresponds to the exact solution. If we restrict ourselves to the first two terms of the expansion (1), we get[4],

$$\Phi_1(\theta)=\frac{x^3}{3k}\left[\frac{k_e-k}{k}+\frac{3\rho_e-3\rho}{2\rho_e+\rho}\cos\theta\right] \tag{3}$$

which is known as the long wavelength approximation (indicated by the subscript l). This approximation is valid for scatterers small in comparison to the wave length of ultrasound. For more complicated particle shapes, we rely on the Green's function approach. In this approach, the wave equation is written in such a way that it accommodates the homogeneous equation as well as an extra term which takes care of the mismatch in density and compressibility and their spatial variation[4],

$$\nabla^2 p-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}\gamma_k(r)+div\left[\gamma_\rho(r)\,grad\,p\right] \tag{4}$$

where

$$\gamma_k(r)=\frac{k_e(r)-k}{k}\qquad \gamma_\rho(r)=\frac{\rho_e(r)-\rho}{\rho_e(r)} \tag{5}$$

inside the scattering region and $y_k = 0$, $y_\rho = 0$ outside the scattering region. If the incident wave is a single frequency wave and the interaction remains time invariant then we may write $p = p_\omega \exp(-i\omega t)$. The equation for the pressure amplitude then becomes,

$$\nabla^2 p_\omega+k^2 p_\omega=-k^2\gamma_k(r)+div\left[\gamma_p(r)\,grad\,p_w\right] \tag{6}$$

After rearranging some terms the equation (6) may be written as,

$$\nabla^2 p_\omega + n^2 k^2 p_\omega = \left[1-\gamma_\rho(r)\right]^{-1} \nabla\gamma_\rho(r).\nabla p_\omega \quad (7)$$

where

$$n^2(r) = \frac{1+\gamma_k(r)}{1-\gamma_\rho(r)} = \frac{k_e^2(r)}{k^2} = \frac{c^2}{c_e^2(r)} \quad (8)$$

with $k=\omega/c$ and $k_e = \omega/c_e$, ω is the angular frequency of the acoustic wave. The angle distribution factor or the scattering amplitude is then expressed as[4],

$$\Phi_s(\theta) = \frac{k^2}{4\pi} \int\left[\gamma_k(r_0)p_\omega(r_0) - i\gamma_\rho(r_0)\frac{a_r}{k}.\nabla_o p_w(r_0)\right] \exp(-iks.r_0)\,dr_o, \quad (9)$$

where $a_r = k_s/k$ is the unit vector along the direction of observer and r_0 is a point within the obstacle. Since we are considering only the elastic scattering, then $|k_s| = |k|$.

It is clear from the equation (9) that the knowledge of pressure field inside the inhomogeneity is sufficient to calculate the scattering amplitude. It it is assumed that,

$$\frac{|k-k_e|}{k} << 1; \quad \frac{|\rho-\rho_e|}{\rho_e} << 1 \quad (10)$$

$$\frac{x|k-k_e|}{k} < 1; \quad \frac{x|\rho-\rho_e|}{\rho_e} < 1 \quad (11)$$

the pressure wave inside the scatterer may be approximated as,

$$p_\omega(r_0) = \exp(-ik.r_0) \quad (12)$$

This is the well known Born approximation. Then by substituting (12) into (9), we get the scattering amplitude as follows,

$$\Phi_b(\theta) = \frac{k^2}{4\pi} \int\left[\gamma_k(r_0) + \gamma_\rho(r_0)\cos\theta\right]\exp(-iq.r_0)\,dr_0 \quad (13)$$

The subscript b stands for the Born approximation. The momentum transfer $k_s - k$ is designated by q. Its magnitude is $2k \sin(\theta/2)$. For a homogeneous spherical scatterer the integration can be done analytically to yield,

$$\Phi_b(\theta) = \frac{x^2}{q}\left[\gamma_k + \gamma_\rho\cos\theta\right] j_1(qa) \quad (14)$$

where a is the radius of the sphere and $j_1(qa)$ is the spherical Bessel function of order unity.

Saxon, in the context of light scattering by a dielectric sphere, modified the Born approximation by introducing a multiplicative factor k_e/k with the wave number k of the incident wave. Essentially the pressure wave within the scatterer becomes, $p_\omega(r_0) = \exp(-ink.r_0)$. In this way, the property of the medium can be incorporated into the incident wave. For forward scattering, this approximation enlarges the validity domain of the Born approximation[5]. The scattering amplitude in this approximation is,

$$\Phi_{mb}(\theta) = \frac{k^2}{4\pi} \int\left[\gamma_k(r_0) + n\gamma_\rho(r_0)\cos\theta\right]\exp(-iR.r_0)\,dr_0 \quad (15)$$

where $R=nk_s-k$ and its magnitude is $R = k\sqrt{1+n^2-2n\cos\theta}$, the subscript mb refers to modified Born approximation. For the case of a homogeneous spherical scatterer of radius a, the integration leads to,

$$\Phi_{mb}(k_s) = \frac{x^2}{R}\left[\gamma_k + n\gamma_\rho\cos\theta\right] j_1(Ra) \quad (16)$$

Further, if we choose n as $3\rho_e/(2\rho_e+\rho)$, the expression for the scattering amplitude becomes,

$$\Phi_h(k_s) = \frac{x^2}{R}\left[\frac{k_e-k}{k} + \frac{3\rho_e-3\rho}{(2\rho_e+\rho)}\cos\theta\right] j_1(Ra) \quad (17)$$

For $a \to 0$, equation (17) reduces to the long wavelength approximation (3) because $j_1(Ra) \cong Ra/3$. As we shall see in numerical comparisons, this approximation results in significant improvement over the Born approximation while keeping the basic simplicity of the Born approximation. We call it a hybrid approximation because it may be looked upon as combination of Born and long wavelength approximation. The subscript h denotes the hybrid approximation.

BACK SCATTERING IN ONE DIMENSION

Consider now an essentially one dimensional problem of the scattering of a plane wave by a homogeneous layer (characterized by density ρ_e and compressibility k_e) of width 2L (placed parallel to the wave-front). The exact reflection coefficient is given by,

$$R_{ex}(k) = \sqrt{\frac{(\alpha^2 - 1)^2 \sin^2 2k_e L}{4\alpha^2 + (\alpha^2 - 1)^2 \sin^2 2k_e L}} \tag{18}$$

The surrounding medium is taken to be a homogeneous loss-less medium of density ρ and compressibility k. This expression is obtained by solving the wave equation both inside and outside the layer and then matching the solutions at the boundary[6]. The subscript ex stands for exact solution.

The wave equation inside the scatterer takes the form[4],

$$\rho_e \frac{\partial}{\partial x}\left(\frac{1}{\rho_e}\frac{\partial p}{\partial x}\right) = \frac{1}{c_e^2}\frac{\partial^2 p}{\partial t^2} \tag{19}$$

The time independent form of this equation is,

$$\rho_e \frac{\partial}{\partial x}\left(\frac{1}{\rho_e}\frac{\partial p}{\partial x}\right) = -k_e^2 p \tag{20}$$

Introducing the concept of elapsed time, dt, defined as

$$dt = \frac{dx}{c_e} = \frac{ndx}{c}, \tag{21}$$

and employing the relationships,

$$ds = cdt = ndx, \tag{22}$$

$$\frac{d}{dx} = n\frac{d}{ds}, \tag{23}$$

Tobocman[3] showed that the equation (20) can be written as,

$$\frac{d^2 p}{ds^2} + k^2 p = \left(\frac{d}{ds} In\,\alpha\right)\frac{dp}{ds} \tag{24}$$

The term occurring in the right hand side of equation (24) can be recognized as the source term. The reflection coefficient with this identification can be written as[3],

$$R(k) = \frac{1}{2ik}\int_{-L}^{L} exp(iks_0)\left[\left(\frac{d}{ds_0} In\,\alpha\right)\frac{d}{ds_0}\right]p(s_0)\,ds_0 \tag{25}$$

Employing the Born approximation,

$$p(s_0) = exp(iks_0) \tag{26}$$

Tobocman *et al*[3] show that the reflection coefficient can be expressed,

$$R_t(k) = (In\,\alpha)\sin(2kL), \tag{27}$$

where the subscript t refers to Tobocman approximation. In deriving (27), Tobocman *et al*[3] assume that the thickness of layer is 2L even in the s-space. This assumption appears to be inconsistent. Equation (22) tells us that with respect to this new variable space, the width of the layers becomes 2nL. This modification gives,

$$R_{mt}(k) = (In\,\alpha)\sin(2k_e L), \tag{28}$$

where the subscript mt corresponds to modified Tbobocman approximation. As we shall see in next section, this modification results in a significant improvement of the result and that the Tobocman and modified Tobocman lead to similar results only for n close to unity.

NUMERICAL COMPARISON

In this section, we test how good are the other approximation methods in comparison to the Born approximation. In three dimensions, the testing is done for scattering by a homogeneous sphere and in one dimension, the testing is done for a homogeneous layer. It is possible to compute the exact solution for both cases. In either case, the surrounding medium is taken to be a homogeneous and loss-less medium. We choose the surrounding medium as a tissue like medium with acoustical properties, ρ = 1050 kgm^{-3} and c = 1540 ms^{-1} and these give, k =4.01577 $\times 10^{10}$ $kg^1 m^2 s$. The frequency of the ultrasound for medical diagnostics is generally taken from 2 MHz to 10 MHz. We take 2 MHz as the frequency of the incident ultrasound for our numerical calculations for which the wave number of the incident wave becomes k = 815.99809 m^{-1}. The acoustic parameters of the scatterer have been taken with the ratios $\frac{\rho_e}{\rho}$ and $\frac{k_e}{k}$ from 0.85 to 1.15. This is equivalent to, being in the regime n = 0.85 to 1.15 and α ≈0.85 to 1.16 on n and α close to 1.

We calculate the exact backward scattering amplitude using equation (1), and equations (3), (14), (16) and (17) yield the corresponding result using various approximation methods. We define,

$$error(\%) = \frac{\left(|\Phi_{ex}|^2 - |\Phi_{approx}|^2\right) \times 100}{|\Phi_{ex}|^2}$$

Figures 1, 2 and 3 show the error contour charts of various approximations having the size parameters x = 0.01, x = 0.1 and x = 1.0, respectively, for the case of scattering by a homogeneous sphere. It is clear from the Figures 1 and 2 that the hybrid approximation is far superior to the Born approximation and the modified Born approximation. As expected, in this region, its predictions are close to the predictions of the long wavelength approximation. For particle with sizes close to x = 1, the hybrid approximation is far superior to the long wavelength approximation and is as good as the Born approximation. Hence the use of hybrid approximation is preferable to the Born approximation in the domain x ≤ 1 and n-1 << 1. However, the Born approximation is noted to have an edge over the hybrid approximation when n is not close to unity.

For one dimensional case we compute equation (18) to find the exact value of reflection

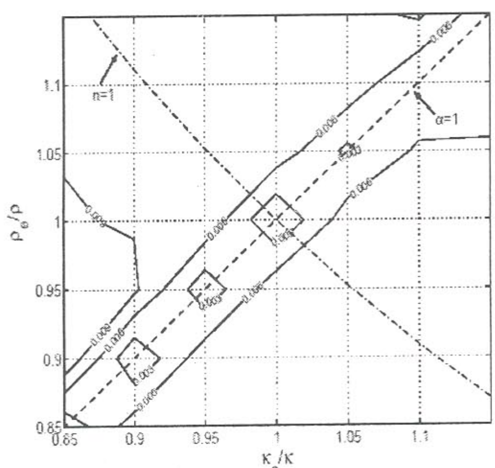

Fig. 1(a). Error contour chart for x=0.01 - in long wavelength approximation

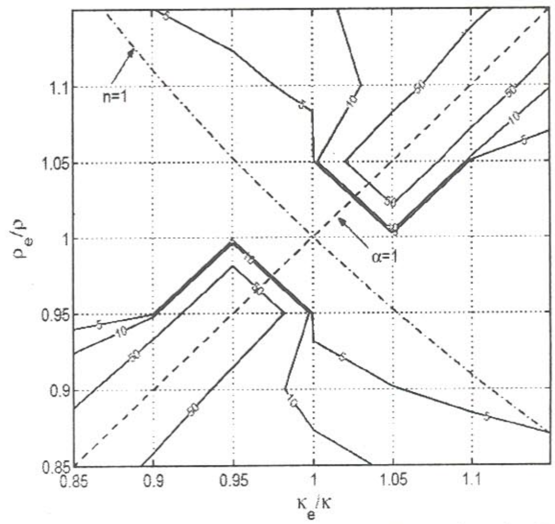

Fig. 1(b). Error contour chart for x=0.01 for the Born approximation

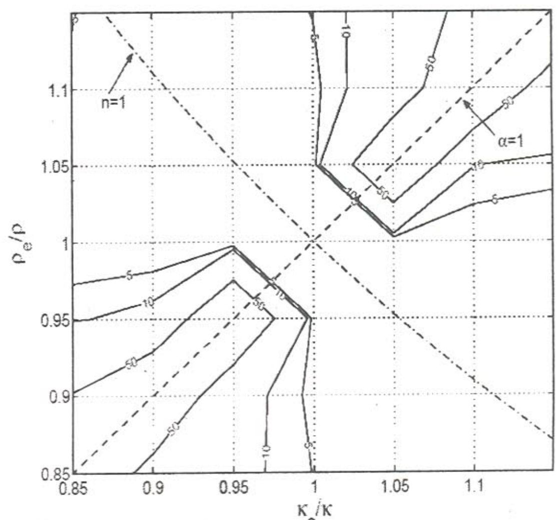

Fig. 1(c). Error contour chart for x=0.01 for the modified Born approximation

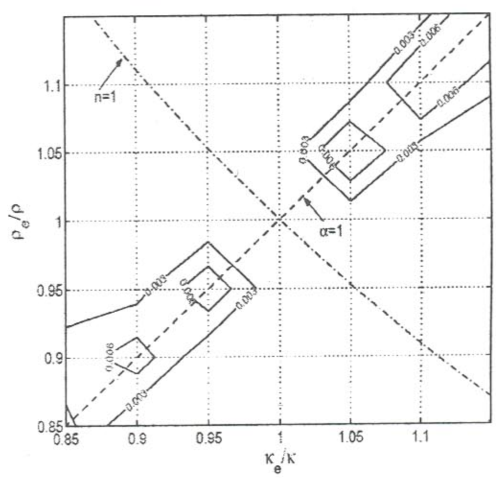

Fig. 1(d). Error contour chart for x=0.01 for the hybrid approximation

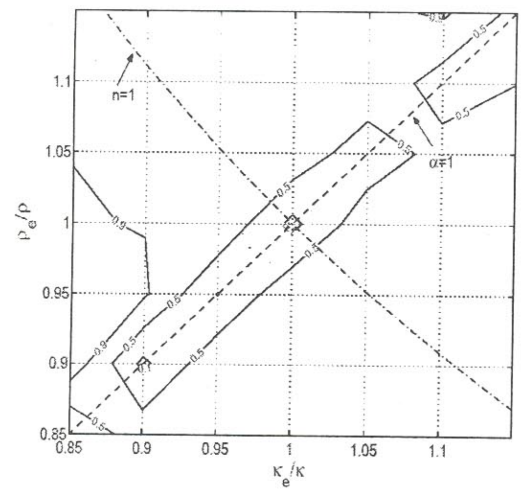

Fig. 2(a). Error contour chart for x = 0.1 - in long wavelength approximation

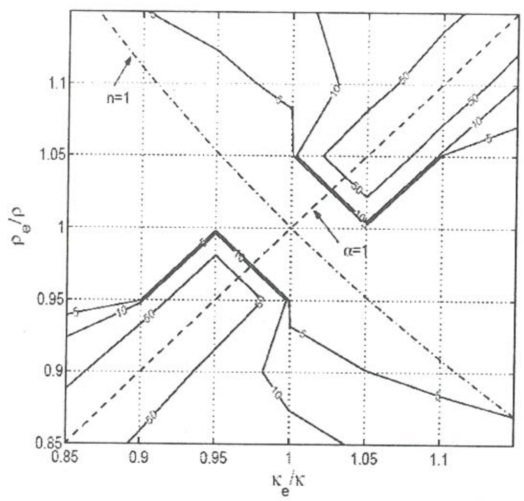

Fig. 2(b). Error contour chart for x=0.1 for the Born approximation

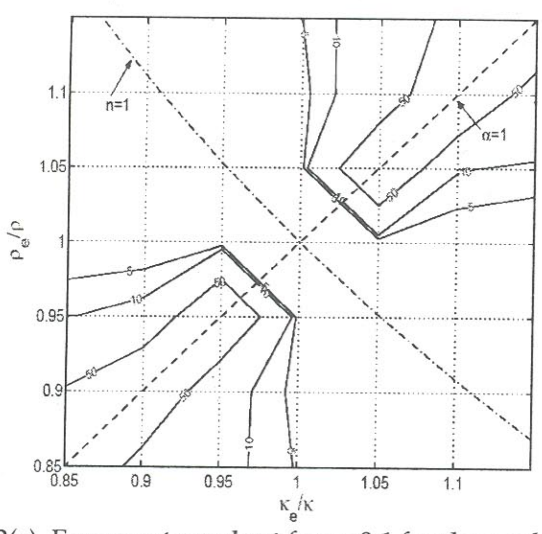

Fig. 2(c). Error contour chart for x=0.1 for the modified Born approximation

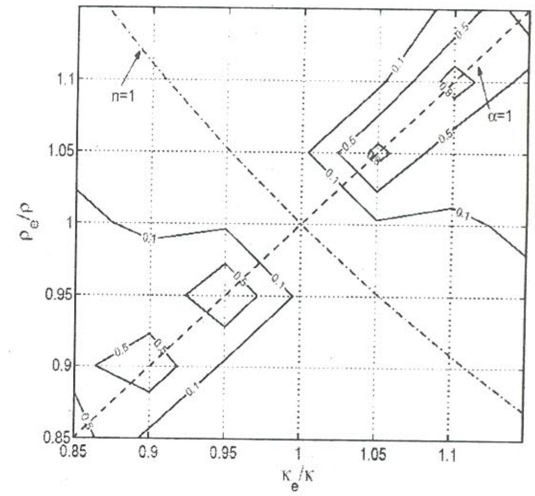

Fig. 2(d). Error contour chart for x=0.1 for the hybrid approximation

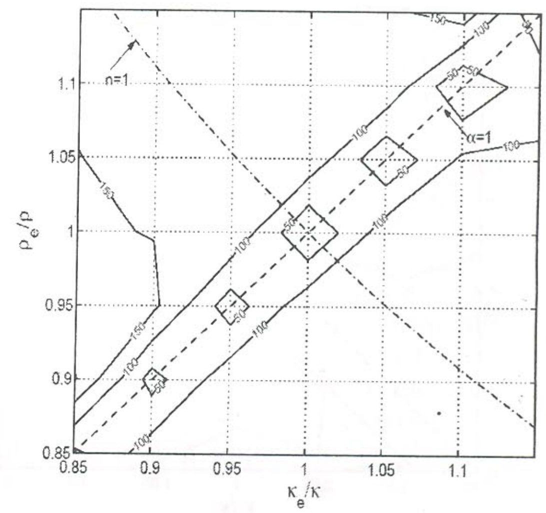

Fig. 3(a). Error contour chart for x = 1 - in long wavelength approximation

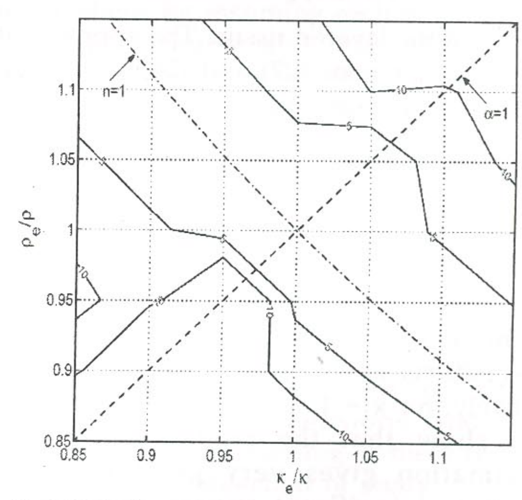

Fig. 3(b). Error contour chart for x = 1 for the Born approximation

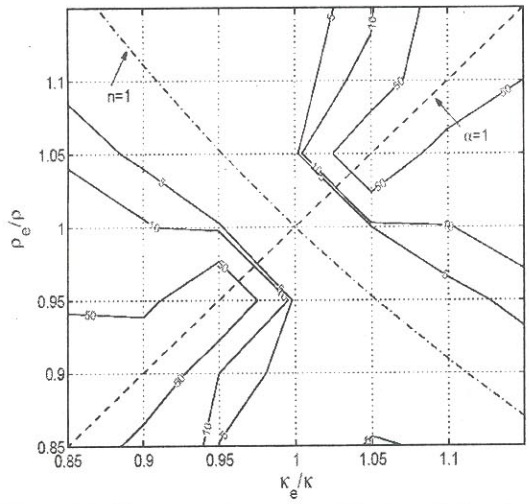

Fig. 3(c). Error contour chart for x = 1 for the modified Born approximation

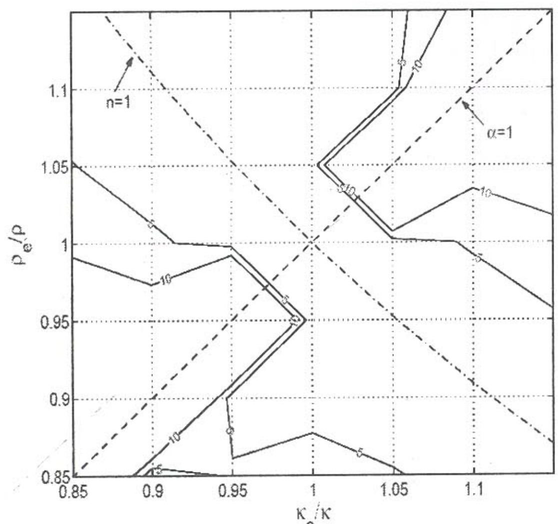

Fig. 3(d). Error contour chart for x = 1 for the hybrid approximation

coefficient of the layered tissue. The approximation results from equations (27) and (28) are compared with the exact results. Percentage error is defined as,

$$error\,(\%) = \frac{\left(|R_{ex}|^2 - |R_{approx}|^2\right) \times 100}{|R_{ex}|^2}$$

The error contour charts for the two approximations are shown in figures 4, 5 and 6 respectively, for x = 1, x = 10 and x = 20. Figures clearly show that the modified Tobocaman approximation gives very good results and constitutes a significant improvement over the Tobocman approximation. Note that the

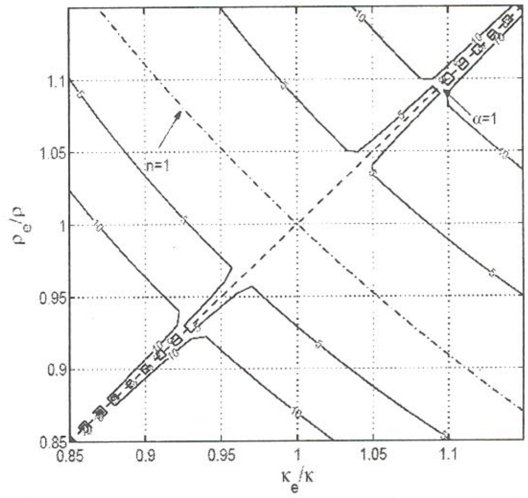

Fig. 4(a). Error contour chart for x = 1 - in Tobocman's approximation

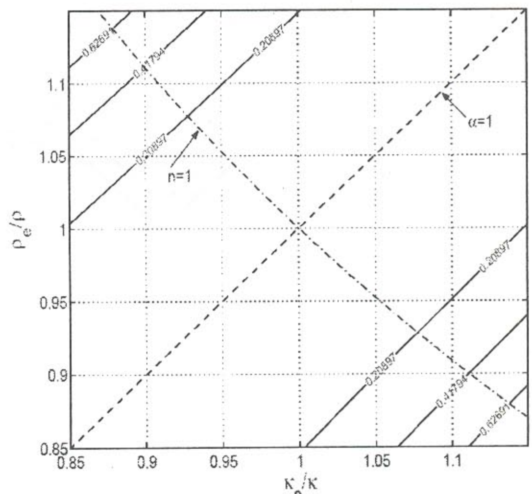

Fig. 4(b). Error contour chart for x = 1 for the modified Tobocman approximation

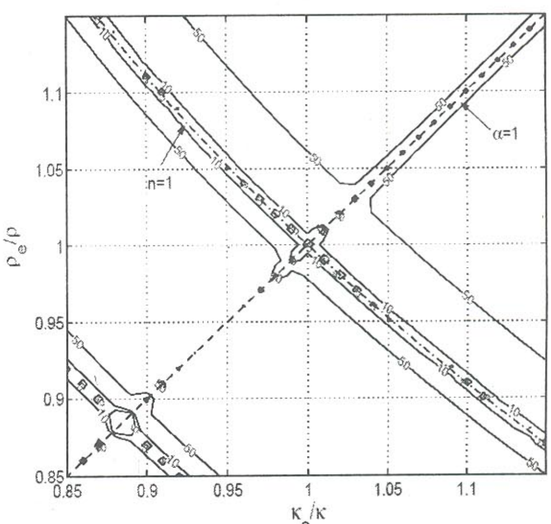

Fig. 5(a). Error contour chart for x=10 - in Tobocman's approximation

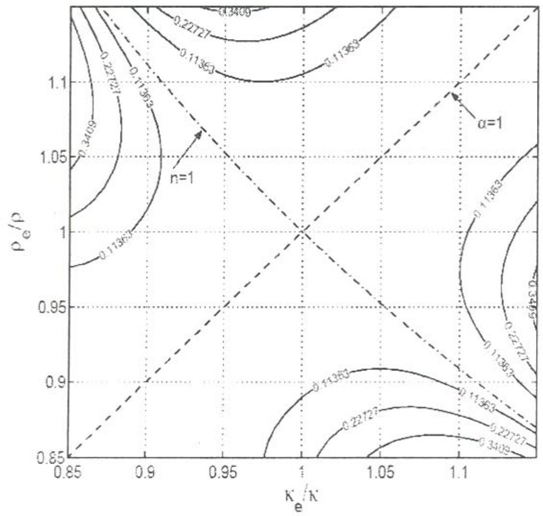

Fig. 5(b). Error contour chart for x=10 for the modified Tobocman approximation

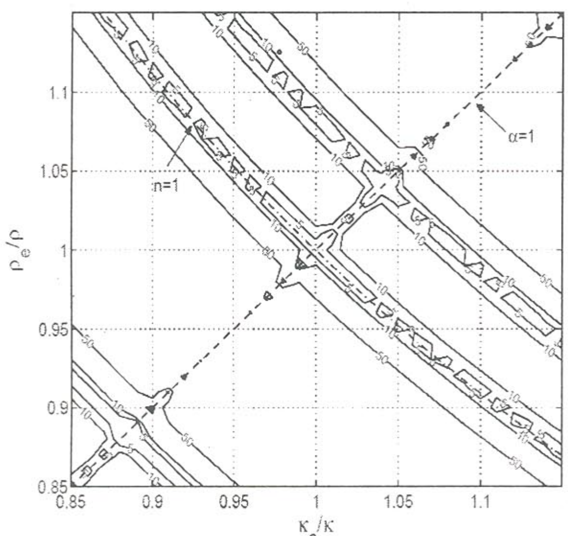

Fig. 6(a). Error contour chart for x = 20 - in Tobocman's approximation

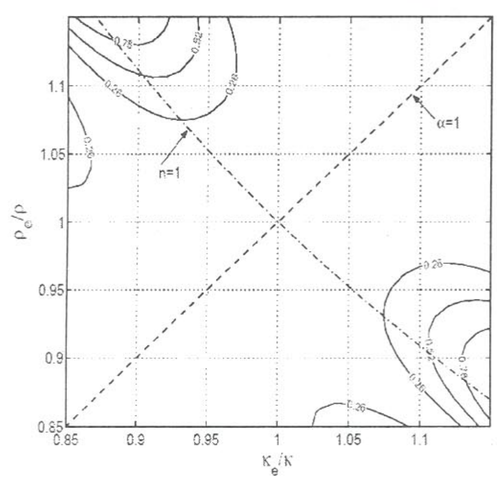

Fig. 6(b). Error contour chart for x = 20 for the modified Tobocman approximation

approximation is good for significantly thick layers too.

CONCLUSION

The aim of the paper was to study quantitatively the accuracy of various approximation methods for the back scattered intensity in three dimensions as well as in one dimension. In three dimensions, we saw that for small values of x the hybrid approximation gives significant improvement over the Born approximation. Hence the hybrid approximation is preferable to the Born approximation for $x \leq 1$ and $n\text{-}1 << 1$. However, for large n values, the Born approximation seems to be the preferable approximation. For values of x close to unity, the hybrid approximation is as good as the Born approximation for n close to unity. For larger values of n, the Born approximation appears to be more suitable for particles of sizes that are of the order of the wavelength of the incident wave.

In case of layered media, the modified Tobocman approximation suggested by us leads to extremely good results. As expected for $n \to 1$, the Tobocman approximation and the modified Tobocman approximation are equally good approximations. The modified Tobocman approximation significantly improves the approximation and surprisingly valid for thickness values well beyond suggested by the theory. Our results show that this approximation should be useful for profiling impedance of thick layers of tissue.

In three dimensions, we calculate the expression of scattering amplitude for different approximations by assuming each point within the scatterer as a source and integrating over the whole inhomogeneity. The physical dimension of the scatterer remains unchanged. Only the pressure field within the scatterer is either taken as the incident pressure field or modified by the acoustic properties of the medium. However, in one dimension as Tobocman *et al* have introduced the concept of the acoustical path, the effect of this is to change the effective physical dimension of the scatterer and the pressure field is assumed as the incident pressure field in the Born approximation. So, our approaches are different in dealing the problems in three dimensions and in one dimension. The understanding of these two spaces bring us to a new direction of study to apply the elapsed time methodology to scattering in three dimensions.

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