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LETTERS TO THE EDITOR

Comments on 'Free of speckle ultrasonic imaging of soft tissue with account of second harmonic signal'

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The Editor,
Sir,
In a recent paper in this journal, Kharin *et al* (2003) have employed a one-dimensional acoustic backscattering method developed by Tobocman *et al* (2002) using Born approximation deconvolved inverse scattering (BADIS), to obtain the impedance profile of a small layered soft tissue structure. Our purpose in this letter is to make a few observations regarding the validity domain of the BADIS method and a simple modification which results in its extension to a wider range of applicability.

We begin by recalling briefly the BADIS formalism (Kharin *et al* 2003, Tobocman *et al* 2002). The starting point is the steady state acoustic wave equation for the excess pressure p':

$$\rho\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial p'}{\partial x} = -\frac{\omega^2}{c^2}p' = -k^2p', \tag{1}$$

where $c = 1/\sqrt{\rho\kappa}$ is the speed of sound and ρ and κ respectively are the density and compressibility of sound in the scattering region. The wave number k is related to the incident wavelength λ, by the relation $k = 2\pi/\lambda$. Introducing the elapsed time $\mathrm{d}t$ such that $c\mathrm{d}t = \mathrm{d}x$, they write

$$\mathrm{d}t = \frac{\mathrm{d}x}{c} = \frac{n\mathrm{d}x}{c_0}, \tag{2}$$

where $c_0 = 1/\sqrt{\rho_0\kappa_0}$ is the speed of sound in water (surrounding medium) and $n = c_0/c$ is the index of refraction. Next a distance $\mathrm{d}s$ is defined such that

$$\mathrm{d}s = c\mathrm{d}t = n\mathrm{d}x. \tag{3}$$

The acoustic wave equation can then be expressed as

$$\frac{\mathrm{d}^2p'}{\mathrm{d}s^2} + k^2p' = \left(\frac{\mathrm{d}}{\mathrm{d}s}\ln z(s)\right)\frac{\mathrm{d}p'}{\mathrm{d}s}, \tag{4}$$

where $z(s) = Z(s)/Z_0$, $Z(s)$ being the acoustic impedance of the layer and Z_0 that of the surrounding medium. Equation (4) is a one-dimensional Schrödinger equation and its solution for the scattering by a layer of thickness $-L$ to L has been previously expressed as

$$R(k) = \frac{1}{2ik}\int_{-L}^{L}\exp(iks)\left[\left(\frac{\mathrm{d}}{\mathrm{d}s}\ln z(s)\right)\frac{\mathrm{d}}{\mathrm{d}s}\right]p_k(s)\mathrm{d}s, \tag{5}$$

where $R(k)$ is related to the impulse response g_{IR} via the relation

$$g_{IR}(s) = \frac{1}{2\pi}\int \mathrm{d}k R^*(k)\mathrm{e}^{iks}. \tag{6}$$

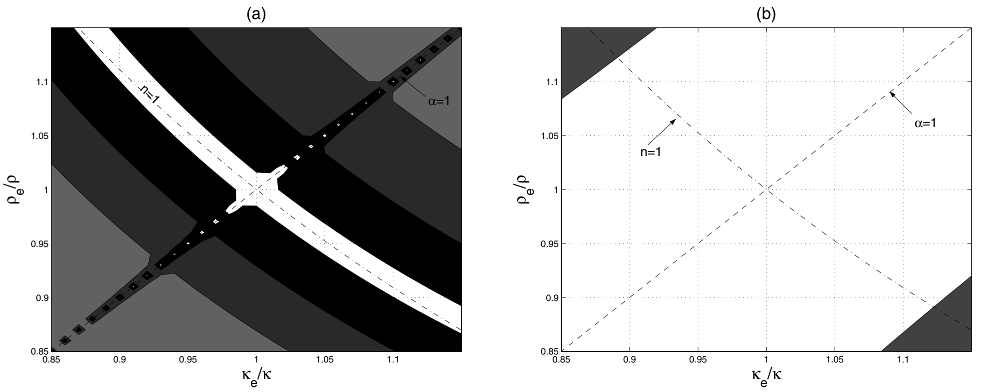

Figure 1. (a) Percent errors in $|R_B(k)|^2$ have been compared for various κ/κ_0 and ρ/ρ_0. The thickness of the layer is such that $\xi = 1.0$. The white region corresponds to errors less than 1 percent. Black: less than 5 percent; less dark shade: less than 10 percent; and the least dark shade: less than 50 percent. (b) Same as (a) but for $|R_{MB}(k)|^2$. White region: errors less than 0.5 percent; dark region: errors between 0.5 and 1.0 percent.

This is the reflected pulse when the incident pulse is a Dirac delta function and $R^*(k)$ is the complex conjugate of $R(k)$. If the scattering is weak so that $p_k(s) \approx \mathrm{e}^{iks}$ in the interaction region, then the reflection amplitude in the Born approximation is

$$R_B(k) = \frac{1}{2}\int_{-L}^{L} \mathrm{d}s\mathrm{e}^{2iks}\frac{\mathrm{d}}{\mathrm{d}s}\ln z(s). \tag{7}$$

Substituting $R_B^*(k)$ for $R^*(k)$ in (6) the final result is

$$z(s) = \exp[4\int_0^s \mathrm{d}y g_{IR}(y)]. \tag{8}$$

The result (8), along with (6), is the basic imaging result of BADIS.

An interesting point in the derivation of (8) is as follows. In writing (5) it was assumed that the thickness of layer, which is $2L$ in x-space, remains the same in s-space too. But according to (3), the width of layer in s-space should ranslate to $-nL < s < nL$. Thus the limits of the integration in (5) should be from $-nL$ to nL. Equation (7), therefore, should be modified to

$$R_{MB}(k) = \frac{1}{2}\int_{-nL}^{nL} \mathrm{d}s\mathrm{e}^{2iks}\frac{\mathrm{d}}{\mathrm{d}s}\ln z(s). \tag{9}$$

The subscript MB refers to the modified Born reflection amplitude. The important thing to note is that the use of this reflection amplitude in (6) also leads to (8).

It is clear from the above discussion that whereas the derivation in Kharin *et al* (2003) and Tobocman *et al* (2002) suggests that the validity of (8) is limited by (7), the modification described here suggests that the validity of (8) is determined by (9). In view of this, it is instructive to examine the validity of approximations (7) and (9) in the case of an exactly soluble model. For this purpose we compare the approximations numerically for the exactly soluble case of a one-dimensional homogeneous layer. We define percent error in the approximations as

$$\text{percent error} = \frac{(|R_{ex}|^2 - |R_{approx}|^2)\times 100}{|R_{ex}|^2}. \tag{10}$$

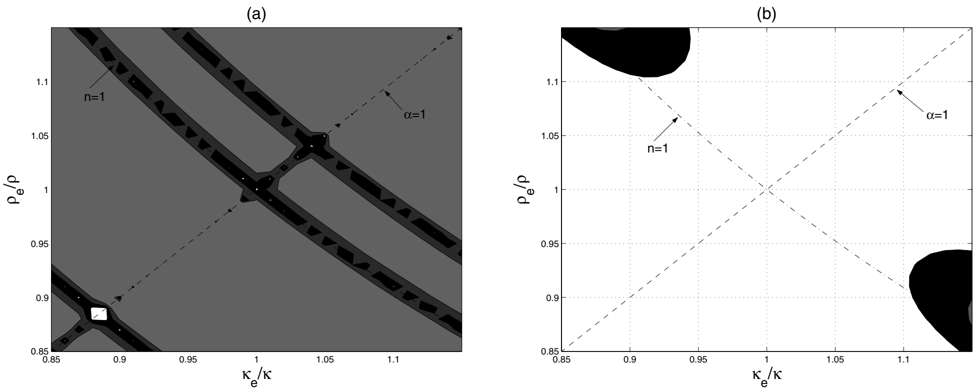

Figure 2. (a) Percent errors in $|R_B(k)|^2$ have been compared for various κ/κ_0 and ρ/ρ_0. The thickness of the layer is such that $\xi = 20.0$. The white region corresponds to errors less than 1 percent. Black: less than 5 percent; less dark shade: less than 10 percent; and the least dark shade: less than 50 percent. (b) Same as (a) but for $|R_{MB}(k)|^2$. White region: errors less than 0.5 percent; dark region: errors between 0.5 and 1.0 percent.

The exact reflection amplitude, R_{ex}, for a homogeneous layer of width $2L$ characterized by density ρ and compressibility κ is given by (Kinsler *et al* 2000),

$$R_{ex}(k) = \sqrt{\frac{(z^2-1)^2\sin^2 2nkL}{4z^2 + (z^2-1)^2\sin^2 2nkL}}. \tag{11}$$

The error contour charts of the two approximations are shown in figures 1 and 2 respectively for $\xi = 1$ and $\xi = 20$. The size parameter $\xi = 2\pi L/\lambda$ can be looked upon as a measure of the size of the particle in terms of the wavelength. The scatterer is weak in the sense that ρ/ρ_0 and κ/κ_0 are close to 1, which is indeed the case for majority of tissues. The figures clearly show that the errors in $|R_B|^2$ can be quite large for certain values of $\rho_e/\rho,\ \kappa_e/\kappa$ even for weak scatterers with thin layers. In comparison the errors in $|R_{MB}|^2$ are negligibly small even for comparatively thick layers.

R K Saha, S K Sen, S K Sharma and B Dutta-Roy,
Saha Institute of Nuclear Physics and
National Centre for Basic Sciences,
Kolkata,
India

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